Appendix A
TABS-MDS Hydrodynamic Description

TABS-MDS (Multi-Dimensional, Sediment) is a finite element, hydrodynamic model. It is based on RMA10, a model written by Dr. Ian King of Resource Management Associates (King 1993). It is capable of modeling turbulent, sub-critical flows using one-, two-, and/or three-dimensional (1-D, 2-D, and/or 3-D) elements. It is also capable of modeling constituent transport including salinity, temperature, and/or fine-grained sediment. The model is capable of coupling the spatial density variation induced by concentration gradients in the constituent field to the hydrodynamic calculations. This enables the model to simulate phenomena such as saline wedges in estuaries. The model has features that permit the simulation of intermittently wetted regions of the domain, such as coastal wetlands.

Theoretical Development

3-D equations

There are six unknowns (u,v,w,h,s,r) and, therefore, six equations are required as follows:

The Navier-Stokes Equations (i.e. conservation of fluid momentum)

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}-\frac{\partial}{\partial x}\left(\varepsilon_{xx}\frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial y}\left(\varepsilon_{xy}\frac{\partial u}{\partial y}\right)-\frac{\partial}{\partial z}\left(\varepsilon_{xz}\frac{\partial u}{\partial z}\right)+\frac{\partial p}{\partial x}-\tau_x=0 \quad \text{(A.1)}$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}-\frac{\partial}{\partial x}\left(\varepsilon_{yx}\frac{\partial v}{\partial x}\right)-\frac{\partial}{\partial y}\left(\varepsilon_{yy}\frac{\partial v}{\partial y}\right)-\frac{\partial}{\partial z}\left(\varepsilon_{yz}\frac{\partial v}{\partial z}\right)+\frac{\partial p}{\partial y}-\tau_y=0 \quad \text{(A.2)}$$

$$\rho \frac{\partial w}{\partial t} + \rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} - \frac{\partial}{\partial x}\left(\varepsilon_{zx} \frac{\partial w}{\partial x}\right) - \frac{\partial}{\partial y}\left(\varepsilon_{zy} \frac{\partial w}{\partial y}\right) - \frac{\partial}{\partial z}\left(\varepsilon_{zz} \frac{\partial w}{\partial z}\right)$$

$$+ \frac{\partial p}{\partial z} + \rho g - \tau_z = 0 \qquad \text{(A.3)}$$

The Volume Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{(A.4)}$$

The Advection-Diffusion Equation

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} - \frac{\partial}{\partial x}\left(D_x \frac{\partial s}{\partial x}\right) - \frac{\partial}{\partial y}\left(D_y \frac{\partial s}{\partial y}\right) - \frac{\partial}{\partial z}\left(D_z \frac{\partial s}{\partial z}\right) \qquad \text{(A.5)}$$

$$-\theta_s = 0$$

The Equation of State

$$\rho = F(s, t) \qquad \text{(A.6)}$$

where:

τ = applied forces (e.g. wind stress, bed shear stress, Coriolis force)

θ_s = salinity source/sink term

Now we reduce the number of unknowns requiring a simultaneous solution from six to three.

Assuming that the influence of vertical momentum on the system is small and may be neglected, Equation A.3 reduces to the following equation:

$$\frac{\partial p}{\partial z} + \rho g = 0 \qquad \text{(A.7)}$$

Equation A.7 is a statement that the vertical pressure distribution is hydrostatic.

Equation A.4 may then be integrated in the vertical direction to yield the following equation:

$$\int_a^{a+h} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) d\eta = -\int_a^{a+h} \frac{\partial w}{\partial z} d\eta = -w_s + w_b \qquad \text{(A.8)}$$

where:
w_s = the vertical velocity at the water surface
w_b = the vertical velocity at the bed

The surface velocity can be expressed as follows:

$$w_s = u_s \frac{\partial(z_b + h)}{\partial x} + v_s \frac{\partial(z_b + h)}{\partial y} + \frac{\partial(z_b + h)}{\partial t} \tag{A.9}$$

Similarly, the bed velocity can be expressed as:

$$w_b = u_b \frac{\partial z_b}{\partial x} + v_b \frac{\partial z_b}{\partial y} + \frac{\partial z_b}{\partial t} \tag{A.10}$$

where:
u_s, v_s = the surface horizontal velocity components
u_b, v_b = the near bed horizontal velocity components
z_b = the bed elevation

Note that by replacing Equations A.3 and A.4 with A.6 and A.8, we recast the equations such that w is present only in the horizontal momentum equations and the advection diffusion equation. It can now be solved in a separate decoupled calculation using the original form of the continuity equation (Equation A.4). This is done by taking the derivative of Equation A.4 with respect to z and solving for w, applying w_s and w_b as boundary conditions.

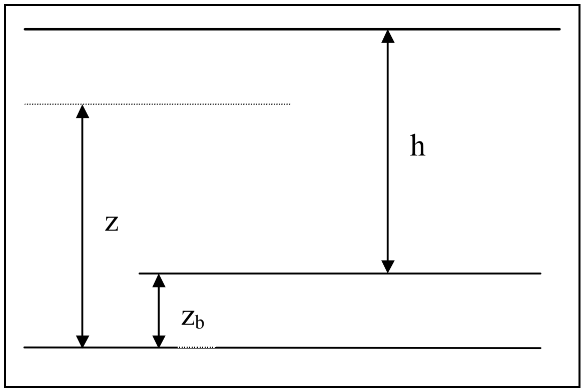
We can further eliminate ρ from the list of unknowns requiring a simultaneous solution by solving the equation of state (Equation A.6) in a decoupled step.

Thus, we are left with four equations (A.1, A.2, A.8, and A.5) and four unknowns (u, v, h, s) to be solved simultaneously. In practice, however, the solution is broken up into twp steps: First the velocities and depth are solved simultaneously, and then the constituent concentration is solved. This method improves solution efficiency dramatically over the simultaneous solution of all four equations and unknowns.

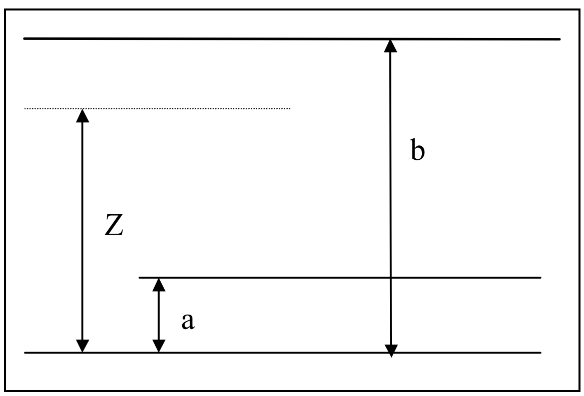
Hence, the solution of a system of four equations and four unknowns becomes the solution of a system of three equations (A.1, A.2, and A.8) and three unknowns (u, v, h), followed by the solution of one equation (A.5) and one unknown (s).

Geometric transform. In order to use a fixed geometry to model a system with a time varying vertical dimension (depth) it is convenient to use a geometric transformation to map the system to a fixed geometry.

Time varying system



Fixed grid system



The transformation is based on the following relation:

$$\frac{h}{(z - z_b)} = \frac{(b - a)}{(Z - a)} \tag{A.11}$$

$$z = \frac{(Z - a)}{(b - a)} h + z_b \tag{A.12}$$

Hence:

$$U(x, y, z) = u(X, Y, \left(\left(\frac{Z - a}{b - a}\right) h + z_b\right)) \tag{A.13}$$

After completing the transformation of the terms and simplifying, we arrive at the following transformed equations:

The Momentum Equations

$$\left\{\begin{array}{l}\rho\left[h\frac{\partial u}{\partial t}+hu\frac{\partial u}{\partial x}+hv\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}(b-a)\left(w-uT_x-vT_y-\frac{(z-a)}{(b-a)}\frac{\partial h}{\partial t}-\frac{\partial z_b}{\partial t}\right)\right] \\ -h\frac{\partial}{\partial x}\left(\varepsilon_{xx}\frac{\partial u}{\partial x}\right)-h\frac{\partial}{\partial y}\left(\varepsilon_{xy}\frac{\partial u}{\partial y}\right)-(b-a)\frac{\partial}{\partial z}\left(\varepsilon_{xz}\left(\frac{(b-a)}{h}\right)\frac{\partial u}{\partial z}\right) \\ +\rho gh\frac{\partial z_b}{\partial x}+\rho gh\frac{\partial h}{\partial x}+h\frac{\partial p}{\partial x}+\rho gh\frac{\partial h_D}{\partial x}-h\tau_x\end{array}\right\}\frac{1}{(b-a)}=0 \tag{A.14}$$

$$\left\{\begin{array}{l}\rho\left[h\frac{\partial v}{\partial t}+hu\frac{\partial v}{\partial x}+hv\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}(b-a)\left(w-uT_x-vT_y-\frac{(z-a)}{(b-a)}\frac{\partial h}{\partial t}-\frac{\partial z_b}{\partial t}\right)\right] \\ -h\frac{\partial}{\partial x}\left(\varepsilon_{yx}\frac{\partial v}{\partial x}\right)-h\frac{\partial}{\partial y}\left(\varepsilon_{yy}\frac{\partial v}{\partial y}\right)-(b-a)\frac{\partial}{\partial z}\left(\varepsilon_{yz}\left(\frac{(b-a)}{h}\right)\frac{\partial v}{\partial z}\right) \\ +\rho gh\frac{\partial z_b}{\partial y}+\rho gh\frac{\partial h}{\partial y}+h\frac{\partial p}{\partial y}+\rho gh\frac{\partial h_D}{\partial y}-h\tau_y\end{array}\right\}\frac{1}{(b-a)}=0 \tag{A.15}$$

Volume Continuity

$$\begin{array}{l}\int_a^b\left[\frac{h}{(b-a)}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\frac{\partial u}{\partial z}T_x-\frac{\partial v}{\partial z}T_y\right]dz \\ +u_s\frac{\partial(z_b+h)}{\partial x}+v_s\frac{\partial(z_b+h)}{\partial y}+\frac{\partial(z_b+h)}{\partial t}-u_b\frac{\partial z_b}{\partial x}-v_b\frac{\partial z_b}{\partial y}-\frac{\partial z_b}{\partial t}=0\end{array} \tag{A.16}$$

Advection-Diffusion Equation

$$\left\{\begin{array}{l} h\dfrac{\partial s}{\partial t}+hu\dfrac{\partial s}{\partial x}+hv\dfrac{\partial s}{\partial y}+\dfrac{\partial s}{\partial z}(b-a)\left(w-uT_x-vT_y-\dfrac{(z-a)}{(b-a)}\dfrac{\partial h}{\partial t}-\dfrac{\partial z_b}{\partial t}\right) \\ -h\dfrac{\partial}{\partial x}\left(D_x\dfrac{\partial s}{\partial x}\right)-h\dfrac{\partial}{\partial y}\left(D_y\dfrac{\partial s}{\partial y}\right)-(b-a)\dfrac{\partial}{\partial z}\left(D_z\left(\dfrac{(b-a)}{h}\right)\dfrac{\partial s}{\partial z}\right)-h\theta_s \end{array}\right\}\frac{1}{(b-a)}=0 \tag{A.17}$$

where:

$$T_x=\frac{\partial z_b}{\partial x}+\frac{(z-a)}{(b-a)}\frac{\partial h}{\partial x}-\frac{h}{(b-a)}\frac{\partial a}{\partial x}+\frac{(z-a)}{(b-a)^2}h\frac{\partial a}{\partial x} \tag{A.18}$$

$$T_y=\frac{\partial z_b}{\partial y}+\frac{(z-a)}{(b-a)}\frac{\partial h}{\partial y}-\frac{h}{(b-a)}\frac{\partial a}{\partial y}+\frac{(z-a)}{(b-a)^2}h\frac{\partial a}{\partial y} \tag{A.19}$$

$$h_D=-\frac{(b-z)}{(b-a)}h \tag{A.20}$$

2-D vertically-averaged equations

If u, v, and s are assumed constant with respect to elevation (z), the 3-D equations can be integrated over depth to yield 2-D vertically averaged equations. For example, the x-momentum equation reduces to the following:

$$\left\{\begin{array}{l} \rho(b-a)\left[h\dfrac{\partial u}{\partial t}+hu\dfrac{\partial u}{\partial x}+hv\dfrac{\partial u}{\partial y}\right] \\ -h(b-a)\dfrac{\partial}{\partial x}\left(\varepsilon_{xx}\dfrac{\partial u}{\partial x}\right)-h(b-a)\dfrac{\partial}{\partial y}\left(\varepsilon_{xy}\dfrac{\partial u}{\partial y}\right) \\ +\rho gh(b-a)\left(\dfrac{\partial z_b}{\partial x}+\dfrac{\partial h}{\partial x}\right)+(b-a)\dfrac{gh^2}{2}\dfrac{\partial \rho}{\partial x}-h(b-a)\tau_x \end{array}\right\}\frac{1}{(b-a)}=0 \tag{A.21}$$

Similarly, the continuity equation reduces to:

$$h\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}+\frac{\partial h}{\partial t}=0 \tag{A.22}$$

And the advection-diffusion equation reduces to:

$$\left\{\begin{array}{l} h(b-a)\dfrac{\partial s}{\partial t} + h(b-a)u\dfrac{\partial s}{\partial x} + h(b-a)v\dfrac{\partial s}{\partial y} \\ -h(b-a)\dfrac{\partial}{\partial x}\left(D_x\dfrac{\partial s}{\partial x}\right) - h(b-a)\dfrac{\partial}{\partial y}\left(D_y\dfrac{\partial s}{\partial y}\right) - h(b-a)\theta_s \end{array}\right\}\frac{1}{(b-a)} = 0 \qquad \text{(A.23)}$$

2-D laterally-averaged equations

Lateral averaging eliminates the momentum equation in the direction normal to the dominant flow direction. The equations are integrated across the width of the channel. This operation requires that the channel width c is specified. For the purposes of TABS-MDS, the channel width in laterally averaged elements is constrained such that it is constant with respect to depth, but can vary with respect to x and y (i.e. along the channel length). For example, the x-momentum equation reduces to the following.

$$\left\{\begin{array}{l} \rho\left[h\dfrac{\partial u}{\partial t} + hu\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial z}(b-a)\left(w - uT_x - \dfrac{(z-a)}{(b-a)}\dfrac{\partial h}{\partial t} - \dfrac{\partial z_b}{\partial t}\right)\right] \\ -h\dfrac{\partial}{\partial x}\left(\varepsilon_{xx}\dfrac{\partial u}{\partial x}\right) - (b-a)\dfrac{\partial}{\partial z}\left(\varepsilon_{xz}\left(\dfrac{(b-a)}{h}\right)\dfrac{\partial u}{\partial z}\right) \\ +\rho gh\dfrac{\partial z_b}{\partial x} + \rho gh\dfrac{\partial h}{\partial x} + h\dfrac{\partial p}{\partial x} + \rho gh\dfrac{\partial h_D}{\partial x} - h\tau_x \end{array}\right\}\frac{c}{(b-a)} = 0 \quad \text{(A.24)}$$

Similarly, the continuity equation reduces to:

$$\begin{aligned} &\int_a^b\left[\frac{h}{(b-a)}\left(c\frac{\partial u}{\partial x} + u\frac{\partial c}{\partial x}\right) - c\frac{\partial u}{\partial z}T_x\right]dz \\ &+ cu_s\frac{\partial(z_b+h)}{\partial x} + \frac{\partial(z_b+h)}{\partial t} - cu_b\frac{\partial a}{\partial x} - \frac{\partial z_b}{\partial t} = 0 \end{aligned} \qquad \text{(A.25)}$$

And the advection-diffusion equation reduces to:

$$\left\{\begin{array}{l} h\dfrac{\partial s}{\partial t} + hu\dfrac{\partial s}{\partial x} + \dfrac{\partial s}{\partial z}(b - a)\left(w - uT_x - \dfrac{(z - a)}{(b - a)}\dfrac{\partial h}{\partial t} - \dfrac{\partial z_b}{\partial t}\right) \\ - h\dfrac{\partial}{\partial x}\left(D_x\dfrac{\partial s}{\partial x}\right) - (b - a)\dfrac{\partial}{\partial z}\left(D_z\left(\dfrac{(b - a)}{h}\right)\dfrac{\partial s}{\partial z}\right) - h\theta_s \end{array}\right\}\frac{c}{(b - a)} = 0 \qquad \text{(A.26)}$$

1-D equations

Under this approximation both vertical and lateral integration are applied. Hence, the form of the cross-section must be defined. In TABS-MDS, the cross section is assumed trapezoidal, with allowance made for off-channel storage. For example, the x-momentum equation reduces to the following:

$$\left\{\begin{array}{l} \rho\left[A\dfrac{\partial u}{\partial t} + Au\dfrac{\partial u}{\partial x}\right] \\ - A\dfrac{\partial}{\partial x}\left(\varepsilon_{xx}\dfrac{\partial u}{\partial x}\right) \\ + \rho gA\dfrac{\partial z_b}{\partial x} + \rho gA\dfrac{\partial h}{\partial x} + \dfrac{gAh}{2}\dfrac{\partial \rho}{\partial x} - A\tau_x \end{array}\right\} = 0 \qquad \text{(A.27)}$$

Similarly, the continuity equation reduces to:

$$A\left(\frac{\partial u}{\partial x}\right) + u\frac{\partial A}{\partial x} + \frac{\partial (A + A_{OC})}{\partial t} = 0 \qquad \text{(A.28)}$$

And the advection diffusion equation reduces to:

$$\left\{(A + A_{OC})\frac{\partial s}{\partial t} + A\frac{\partial s}{\partial x} - A\frac{\partial}{\partial x}\left(D_x\frac{\partial s}{\partial x}\right) - A\theta_s\right\} = 0 \qquad \text{(A.29)}$$

where:
A = The main channel cross-sectional area
A_{OC} = The off-channel storage cross-sectional area

Finite Element Formulation

In order to generate the finite element equations, we must integrate each of the equations over the element volume (for 3-D), area (for 2-D), or length (for 1-D), remembering to include the weight function in the integration (which, for the Galerkin method, is the same as the basis function).

In addition, we must recast the higher-order terms using integration by parts. This causes the boundary terms to drop out of the equations. For example, the following pressure term, multiplied through by a weight function N:

$$N \frac{\rho g h}{(b - a)} \frac{\partial h}{\partial x} \tag{A.30}$$

is then rewritten as:

$$N \frac{\rho g}{2(b - a)} \frac{\partial h^2}{\partial x} \tag{A.31}$$

Equation A.31 can be integrated by parts as follows:

$$\begin{aligned} N \frac{\rho g}{2(b - a)} \frac{\partial h^2}{\partial x} &= \frac{\partial}{\partial x}\left(N \frac{\rho g h^2}{2(b - a)} \right) - \frac{\partial N}{\partial x}\left(\frac{\rho g h^2}{2(b - a)} \right) \\ &- N \frac{g h^2}{2(b - a)} \frac{\partial \rho}{\partial x} - N \frac{\rho g h^2}{2(b - a)^2} \frac{\partial a}{\partial x} \end{aligned} \tag{A.32}$$

Note that the first term on the right hand side of the Equation A.32 can be evaluated as an area integral via the Gauss Divergence Theorem. Hence, it becomes a boundary term.

Time Derivative Solution Method

The time derivative is approximated with a simple, fully-implicit finite difference formulation, i.e.,

$$\frac{\partial \beta_t}{\partial t} = \frac{(\beta_t - \beta_{t-\Delta t})}{\Delta t} \tag{A.33}$$

where:
β_t = any of the unknown variables at time t.
Δt = the time step

Newton-Rhapson Implementation

Once the finite element equations are built, they are solved using the Newton-Rhapson iterative method. In order to do this, partial derivatives with respect to each of the unknown variables must be derived for each system equation. These derivatives compose the stiffness matrix, and are used to drive the residual (i.e. the integral of each equation across an element) to 0.

$$\begin{bmatrix} X_u Y_u Z_u \\ X_v Y_v Z_v \\ X_h Y_h Z_h \end{bmatrix} \begin{bmatrix} u \\ v \\ h \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \text{(A.34)}$$

Applied Loads and Turbulent Mixing

Bed shear stress

The bed shear stress is calculated by a modified form of Manning's Equation, as given by Christensen (1970). Any of three expressions is used depending on the instantaneous value of the depth/roughness height ratio ($\frac{d}{k}$). The expressions are as follows (given for the x-direction only):

$$\text{for } \frac{d}{k} < 4.32 \qquad \tau_x = \frac{\rho\, g}{L^2} \frac{|v| v_x}{d^{2/3}} \quad \text{where } L = \frac{6.46\sqrt{g}}{k^{1/3}} \quad \text{(A.35)}$$

$$\text{for } 4.32 < \frac{d}{k} < 276 \qquad \tau_x = \frac{\rho\, g}{M^2} \frac{|v| v_x}{d^{1/3}} \quad \text{where } M = \frac{8.25\sqrt{g}}{k^{1/6}} \quad \text{(A.36)}$$

$$\text{for } \frac{d}{k} > 276 \qquad \tau_x = \frac{\rho\, g}{N^2} \frac{|v| v_x}{d^{1/6}} \quad \text{where } N = \frac{13.18\sqrt{g}}{k^{1/12}} \quad \text{(A.37)}$$

where:
τ_x = the bed shear in the x-direction
k = the roughness height
d = the local depth
v = the local velocity
g = the gravitational constant

ρ = the density of water
k is found as a function of Manning's n from the following expression:

$$k = \left(\frac{8.25\sqrt{g}\ n}{1.486} \right)^6 \tag{A.38}$$

Wind stress

The wind stress is given by the following expression (given for the x-direction only):

$$\tau_{wx} = \rho_a C_w V_w^2 \cos\theta_w \tag{A.39}$$

where:
τ_{wx} = the wind stress in the x-direction
ρ_a = the density of air
V_w = the wind velocity
θ_w = the direction from which the wind is blowing, measured counterclockwise from the positive X- axis.
C_w = the wind stress coefficient
The wind stress coefficient is given by Wu (1980).

$$C_w = \frac{0.8 + 0.065 \times V_w}{1000} \tag{A.40}$$

Horizontal turbulent mixing and diffusion

Horizontal Turbulent mixing can be specified directly, or it can be controlled by the method of Smagorinsky (1963). A description of this method follows.

The Smagorinsky method of describing horizontal eddy viscosities and diffusion coefficients is a "tensorially invariant generalization of the mixing length type representation" (Speziale 1998). The Smagorinsky description of the turbulent mixing terms in the Navier-Stokes Equations are given as follows. For the x-momentum equation

$$\rho h \frac{\partial}{\partial x}\left(2S \frac{\partial u}{\partial x} \right) + \rho h \frac{\partial}{\partial y}\left(S\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) \tag{A.41}$$

For the y momentum equation

$$\rho h \frac{\partial}{\partial y}\left(2S \frac{\partial v}{\partial y} \right) + \rho h \frac{\partial}{\partial x}\left(S\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) \tag{A.42}$$

where:

$$S = kA\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]^{\frac{1}{2}} \tag{A.43}$$

k = Smagorinsky coefficient, usually given a value ranging from approximately 0.005 for rivers to 0.05 for estuaries and lakes (Speziale 1998; Thomas et al. 1995)
A = the surface area of the element

The Smagorinsky description of the turbulent diffusion terms in the advection-diffusion equation is given as follows:

$$h\frac{\partial}{\partial x}\left(2S\frac{\partial C}{\partial x}\right) + h\frac{\partial}{\partial y}\left(2S\frac{\partial C}{\partial y}\right) \tag{A.44}$$

In order to promote numerical stability, TABS-MDS provides a means of establishing minimum values of turbulent mixing and turbulent diffusion. These values are used in place of the Smagorinsky term (S) when they are found to exceed the value of that term. The minimum turbulent mixing value is given by the following equation:

$$S_{Emin} = \text{TBMINF} \times \rho\alpha\sqrt{A} \tag{A.45}$$

The minimum turbulent diffusion value is given by the following equation:

$$S_{Dmin} = \text{TBMINFS} \times \alpha\sqrt{A} \tag{A.46}$$

where:
TBMINF = minimum turbulent mixing factor (default = 1.0)
TBMINFS = minimum diffusion factor (default = 1.0)
α = a coefficient, given as 5.00×10^{-3} ft/sec or 1.52×10^{-3} m/s, depending on the unit system being used in the simulation. This value is an arbitrary estimate of the minimum turbulent mixing needed to ensure model stability. It equals the value of eddy viscosity/diffusion which corresponds to a Peclet number of 40 and a velocity magnitude of 0.2 ft/sec.

Also, if $|V| <$ TBMINF $\times V_{min}$, S_{Emin} is applied, regardless of the turbulent mixing as given by the Smagorinsky calculation. This is done to inhibit numerical instability in areas with both extremely small velocities and high velocity gradients.

Vertical Turbulent Mixing and Diffusion

Vertical turbulent mixing and diffusion are given by the method of Mellor-Yamada (1982) with a modification according to Hendersen-Sellers (1984).

The Mellor-Yamada expressions for vertical eddy viscosity and diffusion are given as follows:

$$E_{xz} = E_{yz} = \rho S_m l_m q \tag{A.47}$$

$$D_z = S_h l_m q \tag{A.48}$$

where:

$$l_m = 0.4(z-a)\left|1 - \frac{(z-a)}{h}\right|^{\frac{1}{2}} \tag{A.49}$$

$$q = \left\{ b_1 l^2{}_m S_m \left[\left|\frac{\partial u}{\partial z}\right|^2 + \left|\frac{\partial v}{\partial z}\right|^2 \right] \right\}^{\frac{1}{2}} \tag{A.50}$$

where:
$S_m = 0.393$
$S_h = 0.494$
$b_1 = 16.6$

The Henderson-Sellers adjustment is a factor that accounts for the dampening affect on turbulence induced by stable stratification. The factor is expressed in terms of the gradient Richardson Number:

$$R_i = \frac{-g(\partial\rho/\partial z)}{\rho\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]} \tag{A.51}$$

For vertical diffusion of momentum (i.e. vertical eddy viscosity) the expression is given as follows:

$$E_z = \frac{E_{zo}}{(1 + 0.74R_i)} \tag{A.52}$$

Where E_z is the vertical eddy viscosity, and E_{zo} is the vertical eddy viscosity assuming no stratification influence on the turbulence (i.e. the value taken from Mellor-Yamada).

For vertical diffusion of salinity (i.e. vertical diffusion coefficient) the expression is given as follows:

$$D_z = \frac{D_{zo}}{\left(1 + 37R_i^2\right)} \tag{A.53}$$

where D_z is the vertical diffusion coefficient, and D_{zo} is the vertical diffusion coefficient assuming no stratification influence on the turbulence (i.e. the value taken from Mellor-Yamada).